Anomalous fermion bunching in density-density correlation

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We consider theoretically density-density correlation of identical Fermi system by including the finite resolution of a detector and delta-function term omitted in the ordinary method. We find an anomalous fermion bunching effect, which is a quantum effect having no classical analogue. This anomalous fermion bunching is studied for ultracold Fermi gases released from a three-dimensional optical lattices. It is found that this anomalous fermion bunching is supported by a recent experiment (T. Rom *et al* Nature 444, 733 (2006)).

Demonstrated firstly by Hanbury Brown and Twiss [1], quantum noise correlation is a fundamental problem in identical Bose and Fermi system, and has important application in astronomy, quantum optics, condensed matter physics and subatomic physics [2, 3, 4, 5] *etc*. In the last ten years, quantum noise correlation was studied experimentally for cold Bose and Fermi gases [6, 7, 8, 9, 10, 11, 12, 13, 14] with the remarkable development of cold atom physics. Together with these experimental advances, intensive theoretical studies contribute largely to our understanding of correlation effect for cold atomic system. The theoretical studies about high-order correlation have extended from a single harmonically trapped Bose gas [15] to different matter state such as pair correlations of fermionic superfluid [16, 17] and different trapping potential such as high-order correlation of cold atoms in an optical lattice [18]. The finite-temperature effect [19] and low-dimensional effect [20] about high-order correlation were also studied theoretically.

The theoretical studies for fermionic superfluid [16, 17] and one-dimensional ultracold Fermi gas [21] have shown that quantum noise correlation provides important information about the fundamental quantum features of ultracold Fermi gases. Most recently, there is a clear observation of fermion antibunching in a degenerate Fermi gas released from a three-dimensional optical lattice [13]. The fermion antibunching physically arises from the anticommutation relation of field operators, and thus has no classical analogue. It is a manifestation of Pauli exclusion principle in high-order correlation. Besides the observation of the antibunching effect in density-density correlation for fermionic atoms released from an optical lattice [13], fermion antibunching was also observed for electrons [22, 23, 24] and neutrons [25].

In the present Letter, we find that when the resolution of a detector for the measurement of density distribution is considered, there would be an anomalous fermion bunching effect under appropriate conditions. This anomalous fermion bunching effect originates physically from the delta function in the anticommutation relation of fermion field operators, omitted in the ordinary method in calculating the density-density correlation. The consideration of finite resolution of the detector will avoid the notorious divergence in the delta-function term, and lead to anomalous bunching effect. The observation of obvious anomalous fermion bunching effect requests special conditions and parameters. Fortunately, we notice that there is already an anomalous fermion bunching in the experimental data of Ref. [13].

Because we will give a comparison between our theory and experimental data to support the anomalous fermion bunching, we first introduce briefly the elegant experiment in Ref. [13]. In this experiment, an ultracold Fermi gas of ^{40}K was firstly prepared in the combined potential of an optical trap and a 3D (three-dimensional) optical lattice. After free expansion of 10 ms by suddenly switching off the combined potential, 2D (two-dimensional) density images were recorded with a CCD (charge-coupled device) camera by illuminating a resonant laser along the vertical (z) direction. See Fig. 1(a). The 2D density-density correlation was obtained by dealing with appropriately a set of 2D density images.

The starting point of our theory is the 2D density-density correlation function given by

$$C_2(\mathbf{d}, t) = \frac{\int \langle \langle \widehat{n}(\mathbf{r} - \mathbf{d}/2, t) \widehat{n}(\mathbf{r} + \mathbf{d}/2, t) \rangle \rangle_d \, d^2\mathbf{r}}{\int \langle \langle \widehat{n}(\mathbf{r} - \mathbf{d}/2, t) \rangle \rangle_d \langle \langle \widehat{n}(\mathbf{r} + \mathbf{d}/2, t) \rangle \rangle_d \, d^2\mathbf{r}}. \quad (1)$$

Here $\widehat{n}$ is a 2D density operator. $\mathbf{r} \equiv \{x, y\}$ and $\mathbf{d} \equiv \{d_x, d_y\}$. To deal with appropriately the layered distribution of fermionic atoms in 3D optical lattice and absorption imaging along z direction, the 2D density operator takes the form $\widehat{n}(\mathbf{r}, t) = \widehat{g} \widehat{\Psi}^\dagger(\mathbf{r}, t) \widehat{\Psi}(\mathbf{r}, t)$. The operator $\widehat{g}$ has the property that $\left\langle f(\widehat{g}) \widehat{a}_{ij}^\dagger \widehat{a}_{ij} \right\rangle = f(g_{ij})$. $g_{ij} = \sum_{k_z} f_{ijk_z}$ with f_{ijk_z} being the occupation number in the lattice site indexed by $\{i, j, k_z\}$. The summation is about the lattice site along z direction. g_{ij} represents the

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overall particle number in a string of lattice sites along z direction. $\widehat{a}_{ij}$ is an annihilation operator for the atom in an equivalent 2D lattice site indexed by $\{i,j\}$. The 2D density-density correlation function is dependent on $\mathbf{d}$. $C_2(\mathbf{d},t) < 1$ corresponds to the fermion antibunching effect, while $C_2(\mathbf{d},t) > 1$ corresponds an anomalous fermion bunching effect.

We give here a brief introduction about the reason for the above rules in the transformation from 3D system to 2D density-density correlation measured by a CCD. Because the density images were recorded in Ref. [13] with a CCD camera by illuminating a resonant laser along z direction, special consideration should be given about the 2D density-density correlation for a 3D system. The 2D density distribution is

$$
\begin{aligned}
n(\mathbf{r},t) &= \int dz \left\langle \widehat{\Psi}^\dagger(\mathbf{r},z,t)\widehat{\Psi}(\mathbf{r},z,t) \right\rangle \\
&= \Sigma_{ij} g_{ij} \left| \phi_{ij}(\mathbf{r},t) \right|^2 .
\end{aligned}
\tag{2}
$$

Here $\phi_{ij}(\mathbf{r},t)$ is the wave function of the atom initially in the lattice site $\{i,j\}$. Because in 2D density-density correlation, the coordinate z has been integrated, it is convenient to define the 2D density operator as

$$
\widehat{n}(\mathbf{r},t) = \widehat{g}\widehat{\Psi}^\dagger(\mathbf{r},t)\widehat{\Psi}(\mathbf{r},t).
\tag{3}
$$

With the above rules about $\widehat{g}$, $\langle \widehat{n}(\mathbf{r},t) \rangle$ is the same as the result given by equation (2). With this definition of 2D density operator and $\widehat{g}$, it is straightforward to get the final result of the 2D density-density correlation function.

From Eq. (1), it is easy to get

$$
\begin{aligned}
&\langle \widehat{n}(\mathbf{r}-\mathbf{d}/2,t)\widehat{n}(\mathbf{r}+\mathbf{d}/2,t) \rangle \\
&= \left\langle \widehat{g}^2\widehat{\Psi}^\dagger(\mathbf{r}-\mathbf{d}/2,t)\widehat{\Psi}(\mathbf{r}+\mathbf{d}/2,t) \right\rangle \delta(\mathbf{d}) \\
&+ \langle \widehat{n}(\mathbf{r}-\mathbf{d}/2,t) \rangle \langle \widehat{n}(\mathbf{r}+\mathbf{d}/2,t) \rangle \\
&- \left| \left\langle \widehat{g}\widehat{\Psi}^\dagger(\mathbf{r}-\mathbf{d}/2,t)\widehat{\Psi}(\mathbf{r}+\mathbf{d}/2,t) \right\rangle \right|^2 .
\end{aligned}
\tag{4}
$$

The delta-function term in the above formula is ordinarily omitted [13, 26] because the divergent property and $\delta(\mathbf{d}) = 0$ for $\mathbf{d} \neq \mathbf{0}$. This term originates from the delta function in the anticommutation relation $\left\{ \widehat{\Psi}(\mathbf{r}+\mathbf{d}/2,t), \widehat{\Psi}^\dagger(\mathbf{r}-\mathbf{d}/2,t) \right\} = \delta(\mathbf{d})$, and thus accounts for a pure quantum effect.

Bunching (antibunching) corresponds to a peak (dip) in density-density correlation. For $\mathbf{d} \to \mathbf{0}$, the delta-function term in equation (4) means a divergent bunching behavior. When the finite width Δ_d of spatial resolution is considered, it is understandable that there would be an effect of increasing the width and decreasing the height (depth) in the peak (dip) of the bunching (antibunching) behavior. Roughly speaking, $\delta(\mathbf{d})$ may be replaced by a function with height $1/\Delta_d^2$ in the region $-\Delta_d/2 < d_x < \Delta_d/2$ and $-\Delta_d/2 < d_y < \Delta_d/2$, and

with zero value outside this region. When both the delta-function term and spatial resolution are considered, more accurate results are obtained by calculating

$$
\langle \langle \widehat{n}(\mathbf{r}-\mathbf{d}/2,t)\widehat{n}(\mathbf{r}+\mathbf{d}/2,t) \rangle \rangle_d =
$$

$$
\int_\Xi d^2\mathbf{s}_1 \int_\Xi d^2\mathbf{s}_2 \langle \widehat{n}(\mathbf{r}-\mathbf{d}/2+\mathbf{s}_1,t)\widehat{n}(\mathbf{r}+\mathbf{d}/2+\mathbf{s}_2,t) \rangle .
\tag{5}
$$

Here Ξ denotes the region $-\Delta_d/2 < s_x < \Delta_d/2$ and $-\Delta_d/2 < s_y < \Delta_d/2$. In the above formula, $\langle \rangle_d$ represents the average due to the finite resolution of the detector. This average has been considered in the starting point (1).

From equation (1), we get the following approximate result

$$
C_2(\mathbf{d},t) \approx 1 - \frac{\left| \Sigma_{jp} g_{jp} e^{-i2\pi(d_x j + d_y p)/l} \right|^2}{N^2} + \frac{S l^2 l_p^2 \Sigma_{ij} g_{ij}^2}{2\pi \Delta_d^4 \sigma^2 N^2}.
\tag{6}
$$

Here $l = 2\pi \hbar t / m l_p$ with m being the atomic mass and l_p being the spatial period of the optical lattice. S is the area of the overlapping region between Ξ_1 (determined by $-\Delta_d/2 < x < \Delta_d/2$ and $-\Delta_d/2 < y < \Delta_d/2$) and Ξ_2 (determined by $-\Delta_d/2 < x - d_x < \Delta_d/2$ and $-\Delta_d/2 < y - d_y < \Delta_d/2$). N is the total particle number. In equation (6), the second term represents the antibunching behavior. This term is obtained by omitting the effect of spatial resolution and under the condition $l/\sigma > L_0/l_p$ and $l \gg \Delta_d l_p/\sigma$. Here L_0 and σ represent respectively the overall width of the Fermi gas and wavepacket width of an atom in a lattice site before switching off the combined potential. The last term in equation (6) physically originates from the delta-function term in the anticommutation relation of field operators. This term is obtained by considering the spatial resolution and under the condition $l/\sigma > L_0/l_p$.

Using the experimental parameters in Ref. [13] and equation (6), Fig.1b gives $C_2(\mathbf{d},t)$ for flight time of 10 ms. Besides eight dark dots representing antibunching, at the center location we notice a bright dot representing bunching. Fig.1c gives further one of the dark dot, while Fig.1d gives further the bright dot. Eight dark dots are due to the second term in equation (6). This term is -1 for $d_x/l = i$ and $d_y/l = j$ with i and j being integers. At the locations of these dark dots, the last term in equation (6) is zero. The bright dot at the center is due to the last term in equation (6), which is larger than 1 close to $d_x = 0$ and $d_y = 0$. If the last term in equation (6) is not included, there should be a dark dot at the center (see Fig.1e). In Ref. [13], the theoretical model without the last term is used to interpret their experimental data. It is quite interesting to note that eight dark dots (rather than nine dark dots without the consideration of the last term in equation (6)) were observed in Ref. [13] (see Fig.2c and its figure caption in this reference)! In
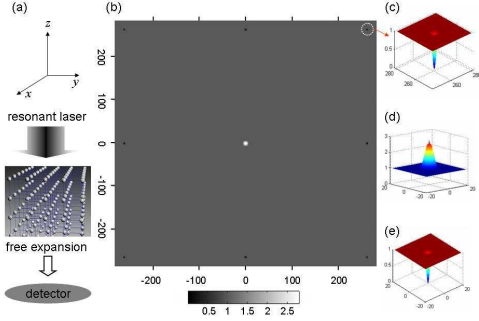
FIG. 1: Fermion antibunching and bunching in density-density correlation function. (a) The measurement of density distribution for a Fermi gas released from a three-dimensional optical lattice. (b) Two-dimensional density-density correlation obtained from equation (6) and experimental parameters in Ref. [13]. The theoretical results agree with the experimental data. In particular, an anomalous fermion bunching is shown clearly by the bright dot at the center, which has been supported strongly in Ref. [13]. (c) Illustration of a dark dot (antibunching effect) in Fig.1b. (d) Illustration of the bright dot (bunching effect). (e) The density-density correlation near the center without the consideration of the delta-function term in the ordinary theory. In these figures, the spatial coordinate is in unit of μm.

Fig.2d of Ref. [13], the obvious and "mysterious" peak at the center of density-density correlation shows further the anomalous fermion bunching behavior. This comparison gives strong experimental evidence for anomalous fermion bunching. It is clear that the condition for observing fermion bunching is $l^2 l_p^2 \Sigma_{ij} g_{ij}^2 > 2\pi \Delta_d^2 \sigma^2 N^2$.

The measurement of density-density correlation extracts the correlation information in quantum noise. We discuss further the physical mechanism of the bunching effect for Fermi system through the consideration of density fluctuations. The density fluctuations are defined as

$$\delta^2 n(\mathbf{x}, t) = \lim_{\mathbf{y} \to \mathbf{x}} \langle \widehat{n}(\mathbf{x}, t) \widehat{n}(\mathbf{y}, t) \rangle - \langle \widehat{n}(\mathbf{x}, t) \rangle^2 . \quad (7)$$

Simple calculations give

$$\delta^2 n(\mathbf{x}, t) = \lim_{\mathbf{y} \to \mathbf{x}} \left\langle \widehat{\Psi}^\dagger(\mathbf{x}, t) \widehat{\Psi}(\mathbf{y}, t) \right\rangle \delta(\mathbf{x} - \mathbf{y}) - \langle \widehat{n}(\mathbf{x}, t) \rangle^2 . \quad (8)$$

We see that there is a divergent δ-function term in $\delta^2 n(\mathbf{x}, t)$. It is obvious that the omission of the δ-function term will lead to absurd result of negative density fluctuations. The divergent δ-function term is a pure quantum effect by noting that it comes from the anticommutation relation between field operators. There is another origin for this δ-function term that the field operators $\widehat{\Psi}(\mathbf{x}, t)$ and $\widehat{\Psi}^\dagger(\mathbf{x}, t)$ comprise infinite modes, rather than only the modes occupied by particles before a measurement. In fact, this property is an essential property of quantum

field theory. The analyses about $\delta^2 n(\mathbf{x}, t)$ show clearly that the δ-function term can not be omitted simply. Similarly to our studies of the density-density correlation, the divergence in the δ-function term can be avoided because the resolution of a detector always has a width. This is equivalent to the presentation that creation or annihilation of a particle at an infinitesimal point is impossible because this would mean an infinite energy exchange.

Note that the anomalous fermion bunching does not violate in any sense the Pauli exclusion principle for identical fermions. The anomalous fermion bunching originates from the delta-function term in the anticommutation relation of field operators. Of course, the Pauli exclusion principle will try to destroy the anomalous bunching effect. The second term in (6) reflects the Pauli exclusion principle, and leads to the antibunching effect, and even can completely destroy the bunching effect at the center.

In summary, our studies give an anomalous fermion bunching, and we have given a strong evidence by analyzing a recent experimental data in Ref. [13]. The condition to observe the anomalous fermion bunching is in fact quite rigorous. It is not surprising that this unique quantum effect is found accidentally in Ref. [13] with highly developed experimental technique. The experimental technique in Ref. [13] gives us chance to test further our theory. (i) The last term in equation (6) increases with the increasing of flight time. Thus, we expect a transition process from the antibunching to bunching at the center of $C_2(\mathbf{d}, t)$, with the increasing of the flight time. (ii) The distribution of fermionic atoms in the lattice sites $\{i, j, k_z\}$ is determined by $i^2 + j^2 + \alpha_z^2 k_z^2 \leq R^2$ at zero temperature. Here α_z is determined by the harmonic potential due to the optical trap. Simple calculations give $\Sigma_{ij} g_{ij}^2 / N^2 \simeq 0.93 (\alpha_z N)^{-2/3}$. We see that decreasing α_z has an effect of enhancing the anomalous fermion bunching effect for identical particle number. This dependence on α_z would give us further chance to test our theory. We believe further experimental studies would deep largely our understanding of the indistinguishability of identical particles, wave packet localization in quantum measurement process and high-order correlation. High resolution of a detector is required to reveal anomalous fermion bunching effect. The astonishing resolution of STM (scanning tunneling microscope) and AFM (atomic force microscope) etc suggests that the experimental and theoretical studies about electrons in an atom or molecule may lead to a new regime about the studies of high-order correlation, quantum noise and quantum measurement etc.

The δ-function term in the anticommutation relation is the direct reason for the anomalous fermion bunching effect. For Bose system, the inclusion of the δ-function term in the commutation relation of field operators may also play important role in density-density correlation. It is easy to understand that the inclusion of the δ-function term will lead to an enhanced boson bunching effect at

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$\mathbf{d} = \mathbf{0}$. In fact, in a recent experiment about the density-density correlation of ultracold bosonic atoms released from an optical lattice, there is a clear enhanced boson bunching effect at $\mathbf{d} = \mathbf{0}$ in $C_2(\mathbf{d}, t)$ (see Fig. 2(c) in [8], where the brightness in the center dot is much higher than other eight bright dots, and this can not be explained without the consideration of the δ-function term.). This sort of enhanced boson bunching effect is also implied strongly in [12]. These analyses show that anomalous fermion bunching effect and enhanced boson bunching effect have common physical origin—the indistinguishability of identical particles and the δ-function term in the anticommutation (commutation) relation of field operators.

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